## **TCSS 435**

## **Homework 2**

**Propositional Logic:**

1. **[8 points]** Translate the following facts into propositional logic sentences. Make sure you list all the propositions and their meanings.
   1. Insects have six legs.
      1. I = Insect
      2. L = six legs
      3. **I🡪S**
   2. If insects had eight legs they would be related to spiders.
      1. I = Insects
      2. L = has eight legs
      3. R = related to spiders
      4. **(I ∧ L) 🡪R**
   3. Animals that are related to Spiders are also related to Scorpions.
      1. A = Animals
      2. S = related to Spiders
      3. C = related to Scorpions
      4. **(A ∧ S) 🡪 C**
   4. If insects are six legged animals then they are not related to scorpions.
      1. I = insects
      2. L = six legged animal
      3. R = related to scorpions
      4. **(I ∧ L) 🡪 ¬R**
2. **[12 points]** Show that the following sentences follow from the given knowledge base using equivalence and inference rule. (Give all the steps of your proof and indicate what rule you used).

**KB :**

* + 1. P ∧ Q
    2. P ⇒ (R ∨ S)
    3. (A ∨ Q) ⇒ (P ⇒ S)
    4. ¬((A ∧ S) ∨ (A ∧ P)) ∨ (Q ∧ R)
    5. A ∨ (¬P ∧ ¬Q)
    6. T RUE ⇒ (A ∧ (P ⇒ Q))
    7. V ∨ ¬P ∨ ¬U

1. P ∧ A
   1. P -I from kb- And elimination of ∧ Q
   2. Q -I from kb- And elimination of P∧
   3. (A ∨ ¬P) ∧ (A ∨ ¬Q) -v- Distribution of ∨ over ∧
   4. A ∨ ¬P -c from a- And elimination of ∧ (A ∨ ¬Q)
   5. A ∨ ¬Q -c from a - And elimination of ∧ (A ∨ ¬P)
   6. ¬P ∨ A -d from a - Commutativity of ∨
   7. P 🡪 A -f from a - implication elimination
   8. A -a,g from a - modus ponens
   9. P ∧ A -a,h from a - introduction of ∧
2. V ⇒ S
   1. A ∨ Q -h from a- introduction of ∨
   2. P 🡪 S -a from b; iii from kb- modus ponens
   3. S -b from b; a from a- modus ponens
   4. S ∨ ¬V -c from b- introduction of ∨
   5. ¬V ∨ S -d from b- Commutativity of ∨
   6. V 🡪 S -e from b- implication elimination
3. T RUE ⇒ Q
   1. Q ∨ ¬TRUE -b from a- introduction of ∨
   2. ¬TRUE ∨ Q -a from c- Commutativity of V
   3. (¬¬)TRUE 🡪 Q -b from c- Implication elimination
   4. TRUE 🡪 Q -c from c- Remove negation double.
4. (Q ∧ U) ⇒ V
   1. ¬P ∨ ¬U ∨ V -vii from kb- Commutativity of V
   2. ¬P ∨ (¬U ∨ V) -a from c- Associativity of ∨
   3. **P 🡪 (**¬U ∨ V) -b from d- Implication elimination
   4. ¬U ∨ V -c from d; a from a- modus ponens
   5. ¬U ∨ V ∨ ¬ Q -d from d- Introduction of V
   6. ¬Q ∨ ¬U ∨ V -e from d- Commutativity of ∨
   7. (¬Q ∨ ¬U) ∨ V -f from d- Associativity of ∨
   8. ¬(Q ∧ U) ∨ V -g from d- de Morgan
   9. (Q ∧ U) 🡪 V -h from d- Implication elimination

**First-order Logic**

1. **[8 points]** Translate the following facts into sentences in first-order logic.
   1. Dogs and cats eat meat.
      1. ∀x∃m(Dog(x) V Cat(x)) 🡪 Eats(x,m))
   2. Every bottle that is filled contains liquid.
      1. ∀x∃l(((Bottle(x) ∧ Filled(x,l)) 🡪(Contains(x,l) ∧ Liquid(l)))
   3. Since Jim and Jack take the same classes, Jack works on the same assignments as Jim.
      1. (∀x(Class(x) 🡪 (Takes(Jim, x) 🡨🡪 Takes(Jack, x)))) 🡪 (∀x(Assignment(x) 🡪  
          (WorksOn(Jim, x) 🡨🡪 WorksOn (Jack, x))))
   4. If Mary is John’s daughter then Mary is younger than John.
      1. Daughter (Mary, John) 🡪 Younger(Mary, John)
2. **[12 points]** Determine for each of the following pairs of sentences if they can be unified and if they can, give the most general unifier.
   1. (Aunt(x, y) ∧ ¬Man(x) ∨ Uncle(x, y))

(Aunt(Mary, z) ∨ ¬Man(John) ∨ Uncle(v, z))

**Can’t be unified. Different variables / syntax.**

* 1. ((Father(x, Jack) ∧ Mother(y, Jack)) ⇒ Married(x, y))

((Father(y, z) ∧ Mother(Mary, z)) ⇒ Married(y, Mary))

**{y = Mary, z = Jack, x = Mary}**

* 1. ((Son(x, x)∧Sister(Mary, Jack)) ⇒ (Daughter(x, Mary)∧Brother(Jack, Mary)))

((Son(Jack, x)∧Sister(z, x)) ⇒ (Daughter(z, f(x))∧Brother(y, z)))

**I don’t think these are unifiable. Jack cannot be the son of himself (the same Jack). Also, I don’t think that you can swap Mary and f(x).**

* 1. ((Married(x, y) ∧ Father(x, Mary)) ⇒ Man(x))

((Married (z, f(Jack)) ∨ Father(z, v)) ⇒ Man(z))

**Can’t be unified. Different variables / syntax. Also, I don’t think you can swap f(jack) and y.**

**Propositional Resolution:**

Resolution rule:

α v β, ¬β v γ

α v γ

Resolution refutation:

* Convert all sentences to CNF
* Negate the desired conclusion (converted to CNF)
* Apply resolution rule until either
  + Derive false (a contradiction)
  + Can’t apply any more

Resolution refutation is sound and complete

* If we derive a contradiction, then the conclusion follows from the axioms
* If we can’t apply any more, then the conclusion cannot be proved from the axioms.

1. **[10 points]** Use resolution with refutation to show that the following queries can be inferred from the given knowledge base.

**KB :**

* + - 1. Father(John, Jack)
      2. Married(John, Jane)
      3. Man(Jack)
      4. Father(x1, y1) ⇒ M an(x1)
      5. Mother(x2, y2) ⇒ Woman(x2)
      6. Married(x3, y3) ∧ Father(x3, z3) ⇒ Mother(y3, z3)
      7. Father(x4 **John**, y4 **Jack**) ∧ Mother(z4 **mary**, y4 **Jack**) ⇒ Married(x4 **John**, z4**mary**) ∨ Divorced(x4 **John**)
      8. Divorced(John) ⇒ False
      9. Mother(Mary, Jack)
      10. Married(x5, y5) ∧ Son(z5, x5) ⇒ Son(z5, y5)
      11. Father(x6, y6) ∧ Man(y6) ⇒ Son(y6, x6)
  1. Married(John, Mary)
     1. ¬Father(John, y4) ∨ ¬Mother(z4, y4) ∨ Married(John, z4)
        1. From KB 7 and KB 8 - x4 = john
     2. ¬Mother (z4, Jack) ∨ Married (John, z4)
        1. From the above b-i and KB 1. - y4= jack
     3. Married(John, Mary)
        1. From b-i and KB 1 - z4 = Mary
     4. **False** (refuted). - from directly above (iii) and ¬query
  2. Woman(Mary) ∧ Woman(Jane)
     1. Woman(Mary)
        1. From kb 5 and 9. - x2 = Mary and y2 = jack
     2. ¬Father(John, z3) ∨ Mother(Jane, z3)
        1. From kb 6 and 2. - x3 = john and y3 = Jane.
     3. Mother (Jane, Jack)
        1. From directly above (ii) and KB 1. - z3 = Jack
     4. Woman(Jane)
        1. From directly above (iii) and KB 5. - x2 = Jane and y2 = Jack
     5. ¬ Woman(Jane)
        1. From clause c-i and ¬query.
     6. **False** (refuted)
        1. From the two directly above (v and iv).